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ELECTROMAGNETS FOR USE WITH 400-CYCLE AIRCRAFT POWER SUPPLY

by

Charles Francis Helme, Jr.

An Essay

Submitted to the Advisory Board

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The School of Engineering

of

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SYMBOLS

A = cross-sectional area

B = flux density

E = volts (r.m.s.)

$\mathcal{E}$ = efficiency of use of core material

f = frequency in cycles per second

F = force

Φ = flux

H = magnetizing force

I = current, amperes

K_h = Steinmetz hysteresis coefficient

K_1, K_n = constants, defined as introduced

L = inductance in henries

l = total length of magnetic circuit

l_a = length of air gap

μ_a = alternating current permeability

μ_e = effective permeability of the magnetic circuit

N = total number of turns in a coil

P_e = total eddy current loss, watts

P_h = total hysteresis loss, watts

R = resistance, ohms

$\mathcal{R}$ = reluctance

ρ = resistivity

t = thickness of laminations

V = volume of magnetic material

W_e = eddy current loss, watts/cu.in.

W_h = hysteresis loss, joules/cu.in./cycle

X = reactance

Subscripts:

m = maximum; B_m, I_m, etc.

4 = frequency of 400 c.p.s.; A_4, l_4, etc.

6 = frequency of 60 c.p.s.; μ_{a6}, N_6, etc.

CHAPTER I

Introduction to the Difficulties of 400-Cycle Electromagnetic Operation

1-1. The Problem of Aircraft Application

The present trend toward use of 400-cycle alternating current on aircraft has created the problem of design of relays and small pull magnets to operate successfully on this supply. To date, the functions of these elements have been performed by direct current magnets. This, of course, requires rectification of the primary alternating current supply, or the use of an auxiliary motor generator.

This paper will examine the difficulties of designing a 400-cycle magnet. Since such a magnet would be intended for aircraft use, the criteria for satisfactory design are reliability of performance, weight and space requirements and cost, initial and operating. Reliability is the prime requisite and with weight and space considerations far overshadows the economic factors.

For comparison purposes and as an alternative, general consideration will also be given to solution of the same problem by use of a direct current magnet and selenium rectifiers.

1-2. Comparison of 400-Cycle and 60-Cycle Operation

As a reference from which to point up the difficulties associated with 400-cycle operation, it will be assumed that if the supply frequency were 60 cycles, a satisfactory magnet would exist. This 60-cycle magnet would exert the required

pull at the given voltage.

The two basic equations obeyed by such a magnet are

$$E = \sqrt{2}\pi f N \Phi_m \times 10^{-8} \text{ volts} \quad (1)$$

$$F = \frac{Bm^2 A}{144} \text{ pounds} \quad (2)$$

where Bm = flux density in kilomaxwells per sq. in.

A = sq. inches of pole area

Equation (1) neglects the resistance voltage which will be negligible in a well-designed alternating current magnet. Equation (2) gives the pull in pounds for flat surfaces of area A. The solenoidal pull of a moving plunger type will not be considered for reasons which will be indicated in 2-1.

From Equation (1) for Φ_m constant, i.e., for the same pull at 60 and 400 cycles

$$E = \sqrt{2}\pi 60 N_6 \Phi_m \times 10^{-8} = \sqrt{2}\pi 400 N_4 \Phi_m \times 10^{-8}$$

From which

$$N_4 = \frac{3}{20} N_6 \quad (3)$$

For the same core material

$$N_4 I_4 = N_6 I_6$$

Therefore,

$$I_4 = \frac{20}{3} I_6 \quad (4)$$

This high exciting current at 400 cycles is the most serious aspect of this problem. The magnitude of I_4 has two immediate effects. First, assuming that the coil resistance changes in the same ratio as the number of turns from Equation (3), then

$$I_4^2 R_4 = \left(\frac{20}{3} I_6\right)^2 \frac{3R_6}{20} = \frac{20}{3} I_6^2 R_6 \quad (5)$$

This represents a substantial increase in copper loss. Further, more copper is required to carry the larger reactive power.

The volt-ampere limitation of the magnet increases linearly with the frequency. This limitation is defined as the change in reactive, or quadrature volt-amperes taken by the magnet due to plunger motion. It may be shown that for 1 inch-pound of mechanical work at 60 c.p.s., a change of 85 volt-amperes is required. At 400 c.p.s., then

$$\text{Volt-Ampere Change} = 85 \times \frac{400}{60} = 567 \ \frac{\text{v.a.}}{\text{in.lb.}}$$

This large input at the beginning of a stroke might affect the regulation of an inadequate power supply. More serious, however, is the fact that the contactors which handle the magnet will be called upon to handle this power. This, of itself, could be sufficient to rule out the 400 cycle magnet for heavy work applications.

The effect of change of frequency may be illustrated by Steinmetz's equation for hysteresis loss

$$W_h = K_h B_m^{\,n} \text{ joules/cu.in./cycle} \qquad (6)$$

and upon the eddy current loss from

$$W_e = \frac{k_e}{\rho} f^2 B_m^{\,2} t^2 \text{ watts/cu.in.} \qquad (7)$$

where B_m is the maximum value ($^{B}m/2$) of the cyclic loop density in webers per square inch, t the thickness of the laminations in inches, and ρ the resistivity of the laminations in ohms in an inch cube.

From (6), assuming the loss per cycle is independent of frequency up to 400 cycles*

---

*A valid assumption, see p. 94, "Properties and Testing of Magnetic Materials", Thomas Spooner, McGraw-Hill, 1927.

$$W_{h4} = \frac{20}{3} W_{h6} \qquad (8)$$

where

$$W_{h6} = W_h \times 60 \quad \text{watts/cu.in.} \qquad (8a)$$

and W_{h4} is similarly defined.

From equation (7)

$$W_{e4} = \left(\frac{20}{3}\right)^2 W_{e6}$$

From the foregoing, it is possible to make the summary shown in Table I. An efficient 60-cycle design would have the copper loss approximately equal to the iron losses, therefore it is apparent that the eddy current losses, although radically increased, would originally represent substantially less than half of the total losses.

	60 c.p.s.	400 c.p.s.
Current	I_6	$6.67\ I_6$
Copper Loss	$I_6^2\ R_6$	$6.67\ I_6 R_6$
Hysteresis Loss	W_{h6}	$6.67\ W_{h6}$
Eddy Current Loss	W_{e6}	$44.2\ W_{e6}$

Table I

Design for 400-cycle operation must be directed toward reducing these to a minimum. This immediately introduces the cost factor for high resistivity, high permeability and thinly laminated core material. These large power losses must be dissipated as heat, therefore, dielectric strength of the coil insulation at higher temperatures must be considered. This temperature rise might necessitate use of one of the more unusual types of insulation such as asbestos, glass or chrome-oxide.

Another inherent disadvantage common to most alternating current magnets is the requirement for a shading coil. Without this coil, the force exerted by the alternating current magnet falls to zero at twice line frequency. With proper shading, the pull never falls below a minimum which is sufficient to prevent armature movement.

Improper shading with resultant armature chatter has several disadvantages. First, the beating of the pole faces and the stresses on the core tend to weaken the physical structure of the magnet. The effect of 400 cycles would be greatly magnified when compared with that at 60 cycles. Secondly, the sensitivity of the ear increases from 120 to 800 cycles (twice line frequency); the chatter, therefore, is much more objectionable[*] and at the higher frequency would be more aptly described as a scream.

The design problem is about the same at 400 c.p.s. as at 60 c.p.s. For proper shading, i.e, to maintain the highest ratio of minimum pull to average pull throughout each cycle, it is necessary to increase the resistance of the shading coil in direct proportion to the increase in frequency[≠]. If this is done, the current in the loop remains unchanged as the frequency increases, since the induced voltage also increases directly with the frequency. The copper loss, however, is increased in proportion further complicating the heating problem at 400 cycles.

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[*] See page 407, "Elements of Acoustical Engineering", H. F. Olson, Chapman & Hall Ltd (1940)

[≠] See page 7, Pennsylvania State College Bulletin, Engineering Experiment Station Series, Bulletin No. 52.

CHAPTER II

Detailed Investigation of the Reluctance of the Magnetic Circuit

2-1. Effect of a Change of Physical Parameters.

For any magnetic circuit,

$$\Phi = \frac{\text{mmf}}{\mathcal{R}} = \frac{4\pi NI}{\mathcal{R}} \qquad (10)$$

For the problem at hand, the flux and the number of turns are fixed by equations (1) and (2), therfore the only way to reduce the exciting current is to reduce the reluctance of the circuit.

With this in mind, the type of magnetic circuit, its shape with its physical dimensions and the choice of a core material will be considered.

The type of circuit employed is not directly dictated by the 400-c.p.s. frequency. In general, however, the moving armature type of magnet is adaptable to problems such as this requiring a relatively large force with a short stroke. This is in contrast to the moving plunger, or solenoid type which is suitable for a longer stroke, but with less force.

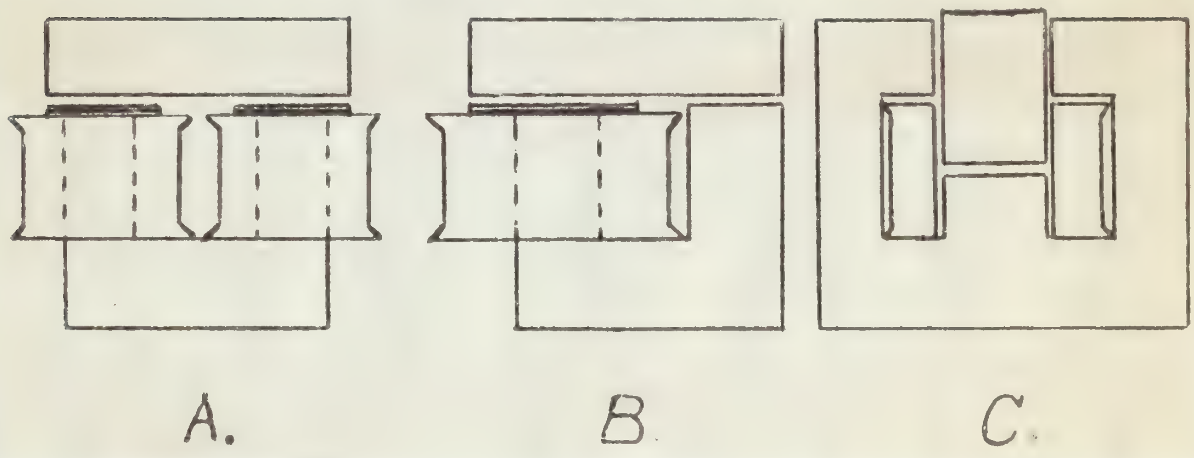


Figure I   Some Core Shapes

Three magnets are indicated schematically in Figure 1. A and B are moving armature types while C is a solenoid type. Of course, many variations of the circuits indicated are in common use, but these three are used to indicate certain guiding principles.

In A and B all the magnetomotive force in the air gaps produces useful mechanical force. For the same number of turns on A and B, in A, the two coil design, pole length may be made half as long as in B permitting a shorter, more efficient magnetic circuit from the viewpoint of leakage. In addition, A possesses a larger radiation surface for cooling.

C, above, must have the fixed gap shown to permit plunger movement. This is a distinct disadvantage because of leakage since the flux loss in this gap decreases the amount of useful flux in the working gap. The fixed and the minimum useful gap lengths here would probably be longer than the two gaps in series in A, or B. For a given useful flux density, therefore, a higher value of magnetizing current would be required in C.

The minimum closed gap length which may be attained is about .001 inches. This is obtainable by plating the finished and fitted surfaces of the laminations with .0005 inches of chromium. Note that this restriction is mechanical, provided the remanence of the core material is small enough to permit the gap length without sticking.

In investigating the physical parameters of the circuit, it will be assumed that the general shape of the core remains unchanged. Further, the rather optimistic closed air gap length

of .002 inches is used in all calculations. This is based upon two .001 inch gaps in series.

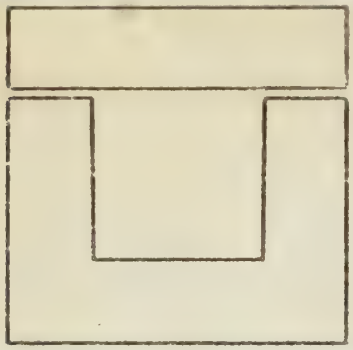


Figure 2. A Magnetic Circuit
Total Length = l
Air gap length = l_a
Cross Section Area = A
Permeability Metal = μ_a

Referring to Figure 2 and neglecting leakage

$$\mathcal{R} = \frac{l}{\mu_e A} \qquad (11)$$

where μ_e is the effective permeability of the circuit which will be discussed at length later. For low reluctance, l should be a minimum. This point need be labored no further. It should be noted, however, that in considering length, the 400 cycle magnet possesses some small advantage over its 60 cycle counterpart. From equation (3)

$$N_4 = 3/20\ N_6 \qquad (3)$$

Since the length of the core is primarily determined by the amount of copper necessary for excitation, a shorter circuit is permitted at 400 cycles than at 60 cycles. The exact decrease in length permitted would be difficult to calculate since some increase in wire diameter is necessary to handle the higher 400-cycle current as indicated in 1-2.

The effect of a change of area will be reflected in the basic equations (1) and (2), therefore it will be investigated in detail. A casual consideration of equation (1) would lead to the conclusion that an increase in area would decrease the reluctance, hence the exciting current

in proportion. An increase of area does decrease the reluctance, but it does not immediately follow that the exciting current is decreased. This will be illustrated by an example.

Given: two 400-cycle magnets both to operate at the same voltage, made of the same core material and to exert the same pull. The general configuration is that indicated in Figure 1, A or B. The characteristics* are

	Magnet #1	Magnet #2
Total length	l	l
Cross Sectional Area	A_1	$A_2 = KA_1$
Maximum Induction	B_{m1}	B_{m2}
Number of Turns	N_1	N_2
Total Gap Length	l_a	l_a

The general equations to be obeyed by both

$$E = \sqrt{2}\pi f N \Phi_m \times 10^{-8} = \sqrt{2}\pi f N B_m A \times 10^{-8} \qquad (1)$$

and

$$F = \frac{B_m^2 A}{144} \qquad (2)$$

Here it is convenient to work in the c.g.s. system of units so that in computing the reluctance of the circuit, advantage may be taken of the fact that the permeability of air is one in this system. Conversion of equation (2) yields

$$F = 4.3 \times 10^{-8} B_m^2 A = 4.3 \times 10^{-8} B_m \times \frac{.4\pi N I_m}{\mathcal{R}} \qquad (2a)$$

where F = pounds
B_m = gauss
A = square centimeters
$\mathcal{R}$ = c.g.s. reluctance units

---

*Subscripts in this problem indicate magnet.

by geometrically [illegible] [illegible] is expressed. This will be illustrated by an example.

Given: Two geometrically similar [illegible] the same voltage [illegible] of the same wire [illegible] the same pulls. The physical configuration is that indicated in Figure 1, 2 or 3. The nomenclature is as follows

	Model 'm'	Prototype 'p'
Small length	l	L
Cross sectional area	A_m	A_p
Ampere turns	NI_m	NI_p
Number of turns	N_m	N_p
Total gap length	l_m	l_p

The general equations to be obeyed by both

$$F = \sqrt{2\pi P}\; I_m \times 10^{-3} = \sqrt{2\pi}\; B_m\; A \times 10^{-3} \qquad (1)$$

and

$$P = \frac{B_m^2 A}{[illegible]} \qquad (2)$$

Here it is convenient to work in the c.g.s. system of units so that in converting the reluctance of the circuit, advantage may be taken of the fact that the permeability of air is one in this system. Conversion of equation (1) yields

$$F = 4.5 \times 10^{-[illegible]} \; [illegible] = 4\pi \times 10^{-[illegible]} \; [illegible] \times \frac{[illegible]}{\mathcal{R}} \qquad (3)$$

where [illegible] = [illegible]
[illegible] = [illegible]
[illegible] = [illegible]
$\mathcal{R}$ = c.g.s. units of [illegible]

[illegible]

The general expression for reluctance is

$$\mathcal{R} = \frac{l - l_a}{\mu_a A} + \frac{l_a}{A} = \frac{l + l_a(\mu_a - 1)}{\mu_a A} \qquad (12)$$

where μ_a = alternating current permeability

Substituting in (2a) and for magnet #1

$$\frac{F}{I_{m1}} = \frac{K_1 \, B_{m1} \, N_1 \mu_a A_1}{l + l_a \, (\mu_a - 1)} \qquad (13)$$

where $K_1 = .4\pi \times 4.5 \times 10^{-8}$

Further from (2) for the same pull and for $A_2 = KA_1$

$$B_{m2} = B_{m1} / \sqrt{K} \qquad (14)$$

From (1) for the same voltage

$$N_1 \, B_{m1} \, A_1 = N_2 \, B_{m2} \, A_2$$

$$= \frac{N_2 \, B_{m1} \, KA_1}{\sqrt{K}}$$

Therefore

$$N_2 = N_1 / \sqrt{K} \qquad (15)$$

For magnet #2, the form of equation (13) is

$$\frac{F}{I_{m2}} = \frac{K_1 \, B_{m2} \, N_2 \mu_a \, A_2}{l + l_a \, (\mu_a - 1)} \qquad (16)$$

and by equations (14) and (15)

$$\frac{F}{I_{m2}} = \frac{K_1 \, (B_{m1}/\sqrt{K})(N_1/\sqrt{K})\mu_a KA_1}{l + l_a \, (\mu_a - 1)} = \frac{F}{I_{m1}} \qquad (17)$$

The ratio of pull to current remains unchanged despite the change of area, therefore for the same pull the same current is required and no direct advantage, i.e., no decrease in exciting current, results.

It might be noted, incidentally, that in the preceding development, the actual frequency did not enter. At any frequency, therefore, for a magnet of given length and core material to exert a given pull at a given voltage, the current is independent of the cross-sectional area. A large area requires less flux density, hence less ampere-turns. To obey the voltage equation, however, less coil turns are necessary and the exciting current remains unchanged.

Although no direct advantage (current decrease) results from an increase of area, certain beneficial secondary effects occur. These are not immediately apparent, but are, nevertheless, of appreciable importance and must be considered.

Leakage has so far been neglected, but must be taken into account in every practical case. An increase in area decreases the leakage coefficient[*] where

$$\text{Leakage coefficient} = \frac{\text{total flux}}{\text{useful flux}}$$

A decrease in the leakage coefficient means less ampere-turns are required for a given useful flux, thus the efficiency of the magnetic circuit is increased.

The number of coil turns must be decreased to maintain the required pull at the given voltage. This will permit some shortening of the core length by the same reasoning followed when reduction of turns was previously discussed.

The hysteresis loss is changed. From

$$W_h = K_h B_m^n$$ ([illegible])

---

[*] See page 144, "Electromagnetic Devices", H. C. Roters, John Wiley & Sons, Inc., 1941

it may be seen that H_m varies with change of area. For the same force, the maximum induction, B_m, must be decreased as shown by equation (14). Offsetting this decrease, however, is the increase in the volume of core material, hence an increase in the hysteresis loss. These two effects will always oppose one another with change of area. The net result for a given change would depend upon the core material (value of K_h) and the value of the exponent, "n", which is in turn dependent upon the range of induction.

In selecting a core material, the following general characteristics are desired:

1) Small residual flux density to permit small air gaps,
2) Low coercive force to permit use of a small alternating current and reduce hysteresis loss,
3) High resistivity to reduce the eddy current losses and, finally,
4) High permeability to decrease the reluctance.

The first three factors need not be considered further. Increased permeability, however, will decrease the reluctance of the magnetic circuit, will not affect the basic equations (1) and (2) and thus will decrease the exciting current.

The unavoidable air gaps in the magnetic circuit of an electromagnet, such as Figure 2, place an upper limit on the value of permeability which may be profitably used. The equation

$$\mathcal{R} = \frac{1 + l_a(\mu_a - 1)}{\mu_a A} \tag{12}$$

permits the definition of effective permeability

$$\mu_e = \frac{1}{\mathcal{R}_A} = \frac{l\mu_a}{l + l_a(\mu_a - 1)}$$

and since $\mu_a \gg 1$

$$\mu_e \doteq \frac{\mu_a}{1 + \frac{l_a}{l}\mu_a} \qquad (18)$$

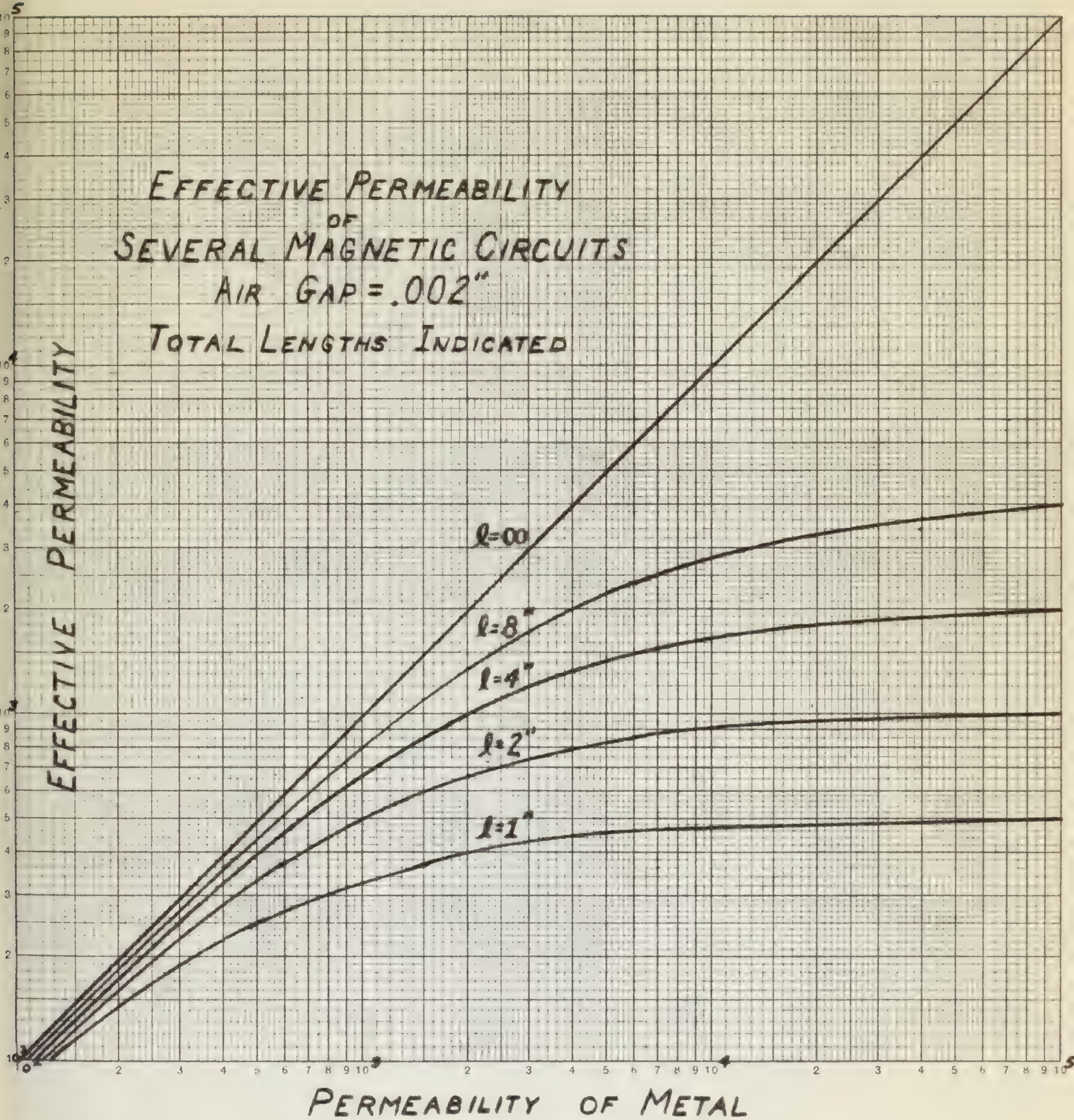
This equation is plotted in the graph of Figure 3 for total lengths indicated each containing a 2 mil air gap. For reference purposes, a line for a circuit of infinite length (or a circuit with no air gap) is also shown. It can be seen that a value of 100,000 yields only slightly better net performance than a value of 10,000, except for l infinite when μ_e equals μ_a. The value of effective permeability asymptotically approached in each case is

$$\lim_{\mu_a \to \infty} \mu_e = \frac{l}{l_a}$$

The graph of Figure 4 follows that of Figure 3 and presents the same information in a more illuminating form. The ratio of

$$\frac{\mu_e}{\mu_a} = \mathcal{E} \qquad (19)$$

is a measure of the efficiency of the use of the core material in a magnetic circuit and is here so defined. A circuit with no air gap would have an efficiency of 100%. The value of $\mathcal{E}$ indicates the return realized by use of metals with progressively higher permeabilities and points up more clearly the limitations to the use of high permeability materials. The impracticality for all


EFFECTIVE PERMEABILITY OF SEVERAL MAGNETIC CIRCUITS
AIR GAP = .002"
TOTAL LENGTHS INDICATED
EFFECTIVE PERMEABILITY
PERMEABILITY OF METAL
l=∞
l=8"
l=4"
l=2"
l=1"
10
5
4
3
2
10
1
2
3
4
5
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3
4
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6
7
8
9

FIGURE 3.

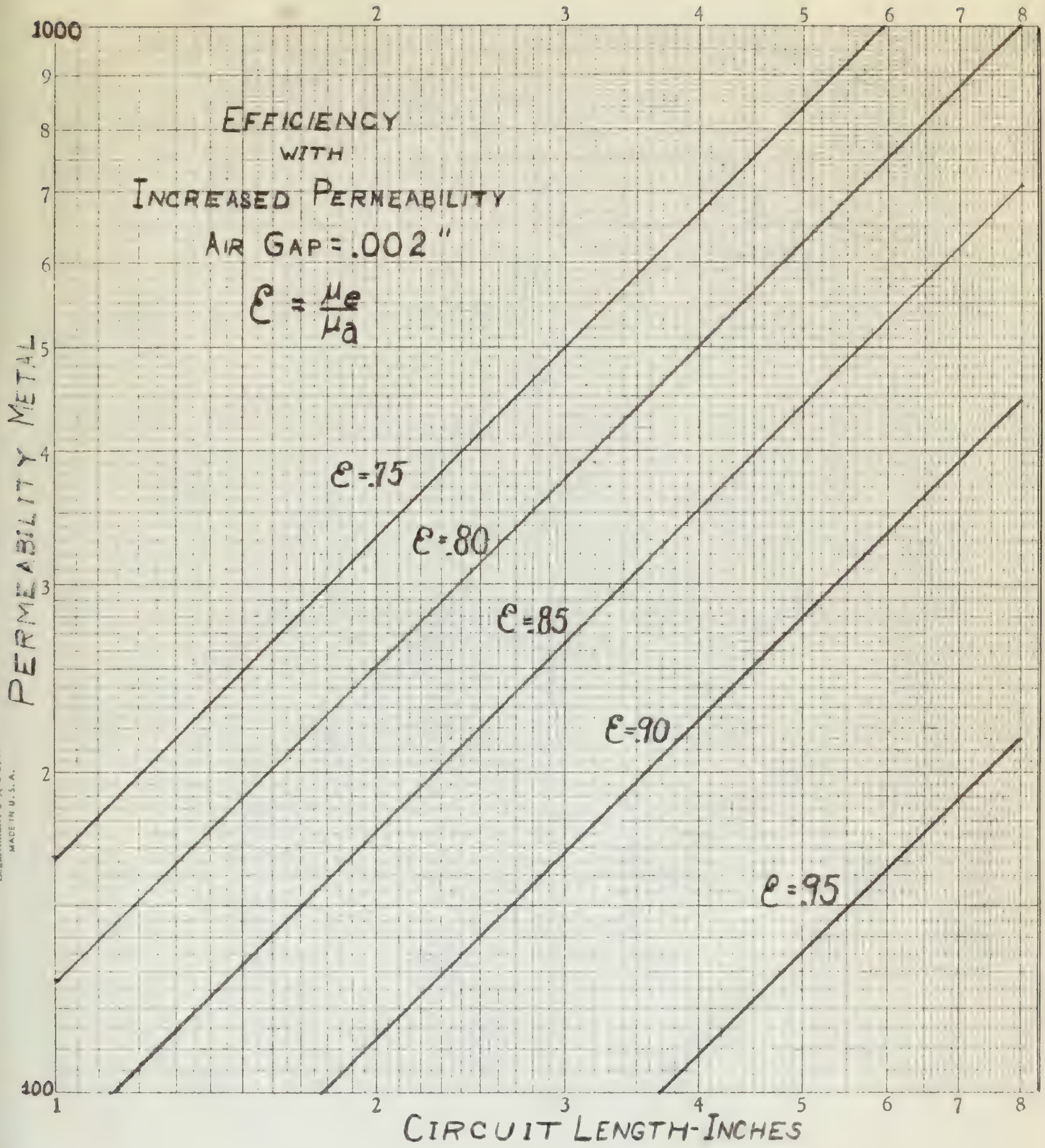

FIGURE 4.

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but the most pressing requirements of five digit (and for shorter lengths, four digit) permeability is clearly illustrated. Even for the eight inch circuit, the efficiency has dropped to 60% when μ_a is 1000 and is down to 90% for a permeability of only 4000. This certainly introduces the economics into this problem.

The primary interest in increasing permeability is, as previously stated, to decrease the exciting current. To examine the possible decrease when permeability is changed, it is necessary to make certain assumptions concerning the reference 60 c.p.s. magnet. First, it is assumed that the core is made of 29 gauge laminations of 4.5% silicon steel. A value of B_m equal 90 kilomaxwells/sq.in. is assigned as a maximum operating induction for this metal which defines the relative permeability as

$$\mu_{a6} = 2300 \text{ gauss/oersted}$$

This is a likely value*.

This magnet is now to exert the same force with the same applied voltage at the 400 c.p.s. frequency. None of the physical dimensions is to be changed, but the number of turns

$$N_4 = \frac{3}{20} N_6 \qquad (19)$$

to comply with equations (1) and (2). The permeability of the core material at 400 cycles will be changed so that

$$\mu_{a4} = K_2 \mu_{a6} \qquad (20)$$

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*See page 66, "Electromagnetic Devices", H. C. Roters, John Wiley & Sons Inc., 1941

For the same flux at both frequencies

$$I_m = \frac{.4\pi N_6 I_{m6} \mu_{e6} A_6}{l_6} = \frac{.4\pi N_4 I_{m4} \mu_{e4} A_4}{l_4} \qquad (21)$$

From which

$$N_6 I_{m6} \mu_{e6} = \frac{3}{20} N_6 I_{m4} \mu_{e4}$$

and

$$\frac{I_{m4}}{I_{m6}} = \frac{20}{3} \frac{\mu_{e6}}{\mu_{e4}} \qquad (22)$$

Substituting for the effective permeability from

$$\mu_e = \frac{\mu_a}{1 + \frac{l_a}{l} \mu_a} \qquad (18)$$

yields

$$\frac{I_{m4}}{I_{m6}} = \frac{20}{3} \frac{\mu_{a6}}{\mu_{a4}} \cdot \frac{1 + l_a \mu_{a4}}{1 + l_a \mu_{a6}} \qquad (23)$$

which by equation (20) becomes

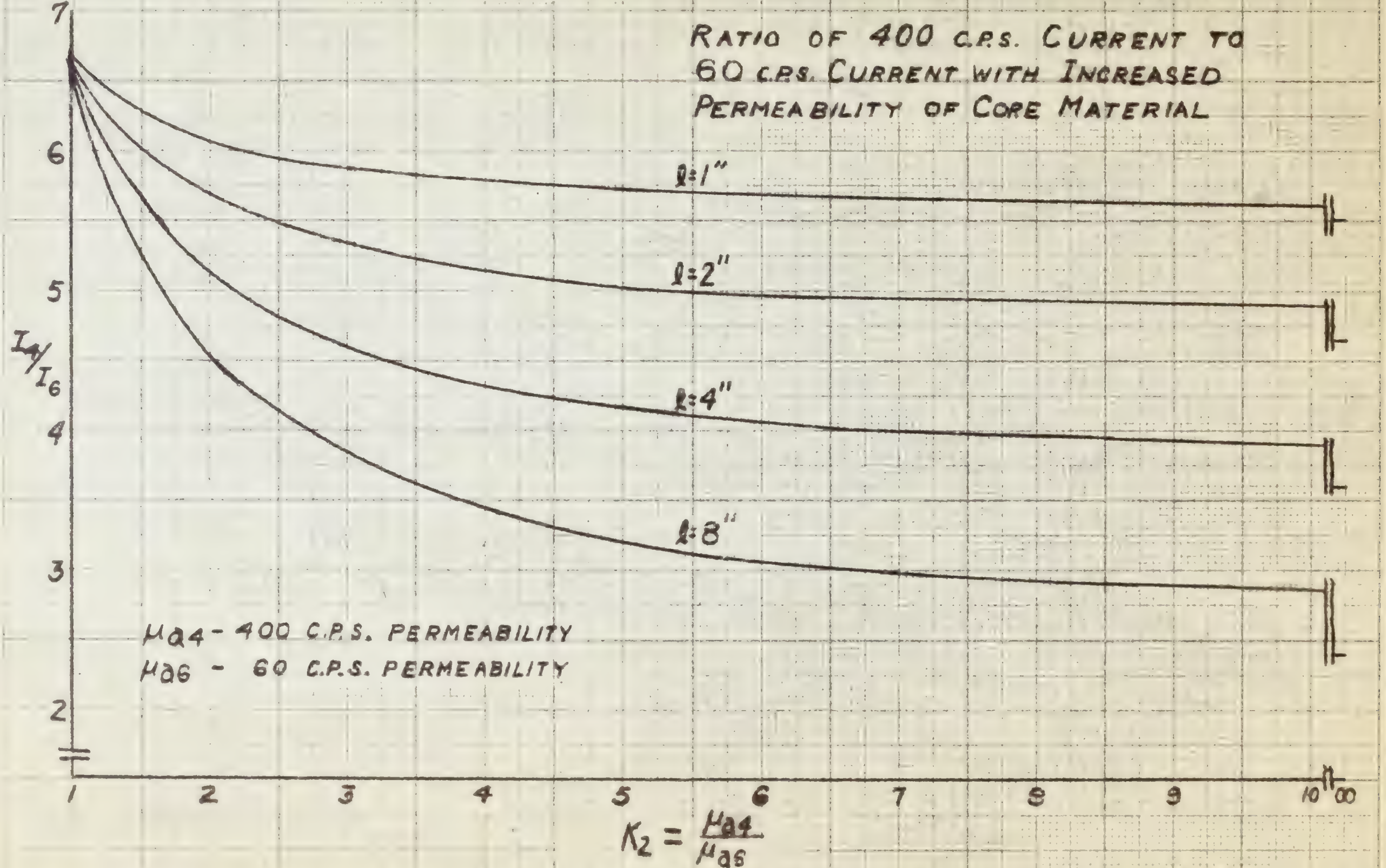
$$\frac{I_{m4}}{I_{m6}} = \frac{20}{3K_2} \cdot \frac{1 + l_a K_2 \mu_{a6}}{1 + l_a \mu_{a6}}$$

Further by the assumption of type of core material and maximum induction at 60 c.p.s. and for an air gap of .002 inches

$$\frac{I_{m4}}{I_{m6}} = \frac{20}{3K_2} \cdot \frac{1 + 4.6 K_2}{1 + 4.6} \qquad (25)$$

where l = mean length in inches.

In Figure 6, equation (25) has been plotted for each of the four circuit lengths considered earlier. K_2 is treated as the independent variable.



RATIO OF 400 C.P.S. CURRENT TO 60 C.P.S. CURRENT WITH INCREASED PERMEABILITY OF CORE MATERIAL

FIGURE 5.

For all lengths when k_p is unity, i.e. for the same magnet core at 60 and 400 c.p.s.

$$\frac{I_{m4}}{I_{m6}} = 6\frac{2}{3} \qquad (4)$$

as expected from section 1-2. Considering again the eight inch circuit, it may be seen that there is little gain in increasing k_p beyond 7.2 and that for this value

$$\mu_4 = 7.2\ (2300) = 16{,}600 \text{ gauss/oersted}$$

and that

$$I_4 = 3I_6$$

For this condition, the efficiency

$$\mathcal{E} = 19.2\%$$

Similar observations may be made for the other circuit lengths.

It may be concluded from this discussion that increased permeability offers some possibility of limiting the high value of exciting current, but that this approach is not very promising, particularly for the shorter circuit lengths.

A secondary effect of increasing permeability is to decrease the hysteresis loss. The lower exciting current decreases the total area of the hysteresis loop, hence the loss associated with that area.

2-2. REDUCTION OF 400-CYCLE LOSSES.

The preceding section leads to the question of how much the various changes discussed can reduce the 400-cycle

losses from what would result with no changes in the magnetic circuit with increased frequency. The 400-cycle losses with no changes are contained in Table I. The principles of 2-1 will be applied and a similar table will be made. As before, it will be assumed that the 60-c.p.s. magnet is exerting a given pull at a fixed voltage; the 400-c.p.s. magnet must exert the same pull for the same voltage. To improve 400-cycle performance one may

1) Go to a shorter length, since less turns are required,
2) Select a core material with a higher permeability, and a lower core loss, $W_h + W_e$,
3) Increase the cross-sectional area of the magnetic circuit, and
4) Decrease the lamination thickness.

Each of these changes will be considered in relation to the copper and iron losses.

1) Shorter length, effect on

a- Exciting Current. From section 1-2 for the same length circuit at 60 and 400 c.p.s.

$$I_4 = \frac{20}{3} I_6 \tag{4}$$

Any decrease in length decreases reluctance in direct proportion with no effect on the basic force and voltage equations. It therefore immediately follows that for shorter length

$$I_4 = \frac{l_4}{l_6} \cdot \frac{20}{3} \cdot I_6 \tag{26}$$

The volt-ampere limitation has been decreased and the contactor problem made less serious.

The copper loss

$$I_4^2 R_4 = \left(\frac{l_4}{l_6} \cdot \frac{20}{3} \; I_6\right)^2 \frac{3R_6}{20} = 6.67 \; \left(\frac{l_4}{l_6}\right)^2 I_6^2 R_6 \qquad (27)$$

and since

$$l_4/l_6 < 1$$

equation (27) may represent an appreciable reduction. This will be further discussed when increased permeability is considered.

b- Hysteresis. Shorter length means less volume. Previously

$$W_{h4} = \frac{20}{3} \; W_{h6} \quad \text{watts/cu.in.} \qquad (8)$$

The new volume

$$V_4 = \frac{l_4}{l_6} \; V_6$$

so that

$$P_{h4} = W_{h4} \; V_4 = \frac{20}{3} \cdot \frac{l_4}{l_6} \cdot W_{h6} \; V_6 \quad \text{watts} \qquad (28)$$

which is again an improvement.

c- Eddy Currents. The above argument for hysteresis holds since total eddy current loss is also dependent upon volume. Therefore,

$$P_{e4} = W_{e4} \; V_4 = \left(\frac{20}{3}\right)^2 \frac{l_4}{l_6} \; W_{e6} \; V_6 \qquad (29)$$

2) Increased permeability, effect on

a- Exciting Current. Assume that the eight-inch

magnet of Figure 4 is the 60-cycle reference and that the permeability is increased to

$$\mu_{a4} = 7.2\,\mu_{a6} = 16{,}600 \text{ gauss/oersted}$$

so that

$$I_4 = 3I_6 \tag{31}$$

Equation (31) holds when the magnetic circuit is the same length at both frequencies. The volt-ampere limitation and the contactor problem are again made less severe. The copper loss now

$$I_4^2 R_4 = (3I_6)^2 \frac{3R_6}{20} = 1.36\, I_6^2 R_6 \tag{32}$$

for an improvement equal to this, equation (27) shows that the 400 c.p.s. mean length must be less than 30% of the 60 c.p.s. mean length if the same core material were used at both frequencies.

b- Hysteresis. The saving in this loss with increased permeability cannot be computed directly, however, it may be stated that there will be a saving. For the same B_m and a lower exciting current, the area of the loop traced each cycle is smaller and the loss decreases directly as this area.

c- Eddy currents. Increased permeability has no effect.

3) Increased cross-sectional area, effect on

a- Exciting Current. There is no direct effect, but the leakage coefficient is improved with an increase of area.

b- Hysteresis. Consider

$$W_h = K_h B_m^n \text{ joules/cu.in./cycle} \qquad (5)$$

and introducing volume

$$P_h = W_h \times V = K_h B_m^n A l \text{ joules/cycle} \qquad (33)$$

Let

$$A_2 = KA_1 \qquad (34)$$

From

$$F = \frac{B_m^2 A}{144} \qquad (2)$$

and for the same force before and after the change of area

$$B_{m2} = B_{m1} / \sqrt{K} \qquad (35)$$

Still not introducing frequency, or the change of length, the ratio of the losses due to an increase of area by a factor K

$$\frac{P_{h1}}{P_{h2}} = \frac{K_h B_m^n A_1 l}{K_h \left(\frac{B_m}{\sqrt{K}}\right)^n KA_1 l} = \frac{1}{K^{1-n/2}}$$

or

$$P_{h2} = K^{1-n/2} P_{h1} \qquad (36)$$

Now let P_{h1} here equal the loss at 400 cycles with shortened length as expressed in equation ([illegible]). The new 400-cycle loss with cross-sectional area increased is

$$P_{h4} = K^{\left(\frac{2-n}{2}\right)} \frac{20}{3} \frac{l_4}{l_6} P_{h6} \qquad (37)$$

The exponent, "n", must be calculated from experimental data for the core material in question; its order of magnitude, however, is approximately 1.5 to 2.0.

Since $K^{\left(\frac{2-n}{2}\right)}$ will always be slightly greater than unity, while l_4/l_6 will always be slightly less, the effects of shorter length and increased area are not appreciable when considered together. Of greater importance is increased permeability which permits writing

$$P_{h4} < \frac{20}{3} P_{h6}$$

c- Eddy Currents. In considering increased area here, the effect of lamination thickness will be covered at the same time.
Referring to

$$W_e = \frac{K_e}{\rho} f^2 B_m^2 t^2 \quad \text{watts/cu.in.} \qquad (7)$$

it was previously shown that for the change of frequency from 60 to 400 cycles

$$W_{e4} = \frac{K_e}{\rho}\left(\frac{20}{3} f_6\right)^2 B_{m6}^2 t_6^2 \quad \text{watts/cu.in.} \qquad (38)$$

Increase of area by a factor K, dictates a decrease in B_m by $1/\sqrt{K}$ for the same force. Suppose that in addition, it is assumed that the lamination at 60 c.p.s. were 29 gauge thickness and that the 400-cycle thickness is

$$t_4 = .006 \text{ inches}$$

about the minimum practical thickness for this application, and a decided improvement over the 60-cycle thickness of .014 inches. Substituting in equation (38)

$$W_{e4} = \frac{K_e}{\rho}\left(\frac{20}{3} f_6\right)^2 \frac{B_{m6}^2}{K} \left(\frac{.006}{.014}\right)^2 t_6^2$$

or

$$W_{e4} = \frac{8.1}{K} W_{e6}$$

but introducing volume

$$P_{e4} = W_{e4} V_4 = \frac{8.1}{K} W_{e6} K V_6 = 8.1 P_{e6} \qquad (39)$$

The improvement is solely the result of thinner laminations.

For comparison purposes, Table II has been prepared. The first three columns are those of Table I. Column 4 represents the conditions attained in this attempt to reduce the 400-cycle losses to a minimum. It must be kept in mind that the circuit length for the magnet represented here is eight inches. If the lengths considered, the improvement indicated in Column 4 is the best obtainable. If a well-designed shading coil were incorporated in both magnets, the 400-cycle copper loss would equal 6.67 times the 60-cycle copper loss for this coil as previously pointed out.

(1)	(2)	(3)	(4)
	60 cps	400 cps same circuit as 60 cps	400 cps losses reduced
Current	I_6	6.67 I_6	3 I_6
Copper Loss	$I_6^2 R_6$	6.67 $I_6^2 R_6$	1.35 $I_6^2 R_6$
Hysteresis Loss	P_{h6}	6.67 P_{h6}	<6.67 P_{h6}
Eddy Current Loss	P_{e6}	44.2 P_{e6}	8.1 P_{e6}

TABLE II

Also, in summation is the fact that the volt-ampere limitation and the contactor problem are less severe with the lowering of the 400-cycle current from 6.67 I_6 to 3 I_6.

CHAPTER III

COMPARISON OF THE ALTERNATING CURRENT-DIRECT CURRENT SOLUTIONS

3-1. The Alternating Current Versus the Direct Current Magnets.

Consideration so far has been limited to solution of a pull problem by direct use of a 400-cycle magnet. From the preceding chapters, it may be concluded that an efficient design of such a magnet would be a most difficult task (considering the 60-cycle reference efficient). In view of this, it appears wise at this point to begin consideration of the alternative direct current solution.

The direct current magnet possesses several attractive advantages over any alternating current magnet, even without the added complication of 400-c.p.s. frequency. In the following discussion, both the alternating current and the direct current magnets will be considered to be operating at the same maximum flux density, therefore the comparison will be somewhat optimistic concerning alternating current operation. For efficient operation, the direct current magnet should operate at saturation; the alternating current magnet should operate below saturation[*].

For the same pole face area, the force of a direct current magnet is twice that of an alternating current

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[*] See page 131, "Electromagnetic Devices", H. C. Roters, John Wiley & Sons, Inc. 1941.

being

$$F = \frac{B^2 A}{72} \quad \text{pounds} \tag{40}$$

where B = flux density, kilomaxwells/sq.in.
A = pole face area, sq.in.

Following immediately from this one may see that for a given pull to be exerted, the alternating current magnet must have twice the cross-sectional area, hence twice the volume and weight required for a direct current magnet. This factor of two represents an absolute minimum. In section 2-2, it was pointed out that a larger area means a lower leakage coefficient. Further, to limit the eddy current losses to a reasonable value at 400 c.p.s., it was shown that laminations should be thin which means a low stacking factor. Both of these recommendations mean greater core bulk at 400 c.p.s. than at 60 c.p.s. for which the volume must already be twice as great as for a direct current magnet for the same pull. The alternating current design will require higher reactive power, hence a greater weight of copper.

In the laboratory, a Price Brothers Type 110 alternating current relay was tested on a 60-cycle voltage. This magnet weighed 32.5 ounces and occupied a volume of approximately 4.1 cubic inches. The volt-ampere consumption was 110 v.a. in the open gap and 15.21 v.a. in the sealed positions. For the same volt-ampere consumption, a direct current magnet could be made with about one half the above weight and volume. In addition, the volt-ampere limitation does not exist and the contactor

problem is less severe in the direct current magnet.

Finally, the alternating current magnet because of the expensive core material (high permeability) and thin laminations which must be used, plus the necessity for a shading coil, is inherently an expensive device. After first cost, because of lower losses, the direct current magnet possesses a continuous advantage in operation.

3-2. The Selenium Rectifier

Tending to offset the advantages of the direct current magnet just listed, is the necessity to rectify the 400-cycle supply for the problem under consideration. The selenium, dry disk rectifier will here be considered for the conversion device.

This rectifier contains no moving parts. It is rugged and requires no routine maintenance. Manufacturers guarantee against any defects whatsoever for periods of 40,000 hours continuous operation, so the aging effects of these devices may be considered negligible. The standard rectifier unit may be hermetically sealed, thus making it impervious to the effects of moisture, fungi and change of pressure. It will operate perfectly at plate temperatures from -40° to 194° F. which should be adequate for the case at hand. This maximum temperature limitation sometimes prevents use of these devices, but in an aircraft application, a plentiful supply of cooling air would be available to overcome this objection. The selenium rectifier is able to withstand heavy temporary

current overloads without danger of breakdown or injury, permitting regular use in the overload condition in intermittent service. Assuming a half-hour period for cooling between overload operations, the rectifier may be operated at ten times normal rating for about ten seconds. The maximum temperature restriction previously given cannot be avoided, however.

Some approximate weight, space and cost data for full wave, bridge type selenium rectifiers are contained in the graphs of Figures 6, 7 and 8. These characteristics are an average of manufacturers data for standard units over the range indicated. For a specific application, the weight and space requirements could undoubtedly be improved upon.

3-3. CONCLUSIONS

The most important consideration in selecting a solution to this problem is, as previously stated, reliability. The characteristics of the selenium rectifier lead to the conclusion that it would be at least, if not more, reliable than the direct current magnet with which it would be associated. Considering the direct current and alternating current magnets themselves equally reliable in operation, the combination of the direct current magnet with a selenium rectifier is equally as reliable in performance of a given task as an alternating current magnet operating directly.

As to weight and space requirements, consider

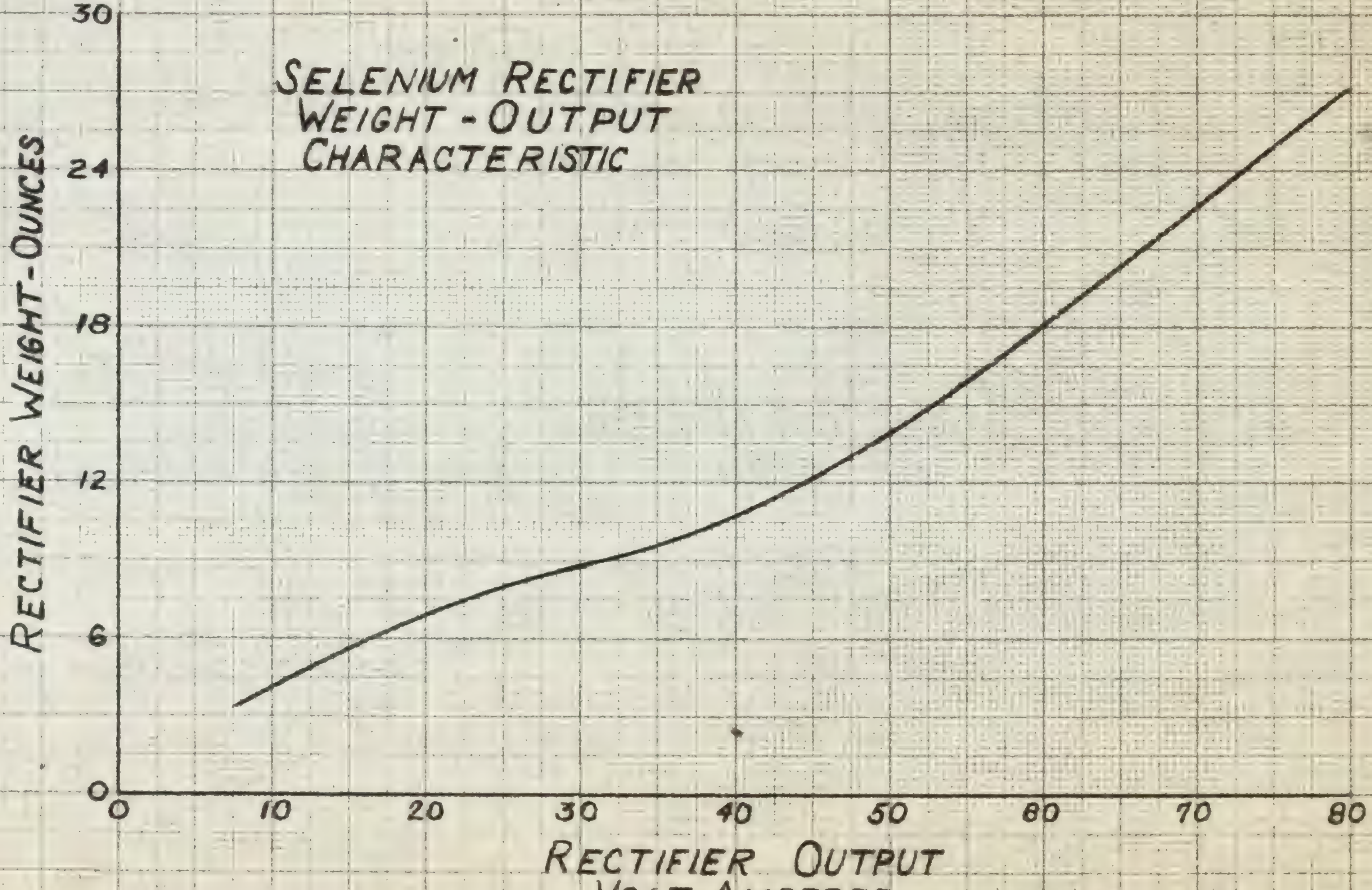
SELENIUM RECTIFIER
WEIGHT - OUTPUT
CHARACTERISTIC
RECTIFIER WEIGHT-OUNCES
30
24
18
12
6
0
0
10
20
30
40
50
60
70
80
RECTIFIER OUTPUT
VOLT-AMPERES

FIGURE 6.

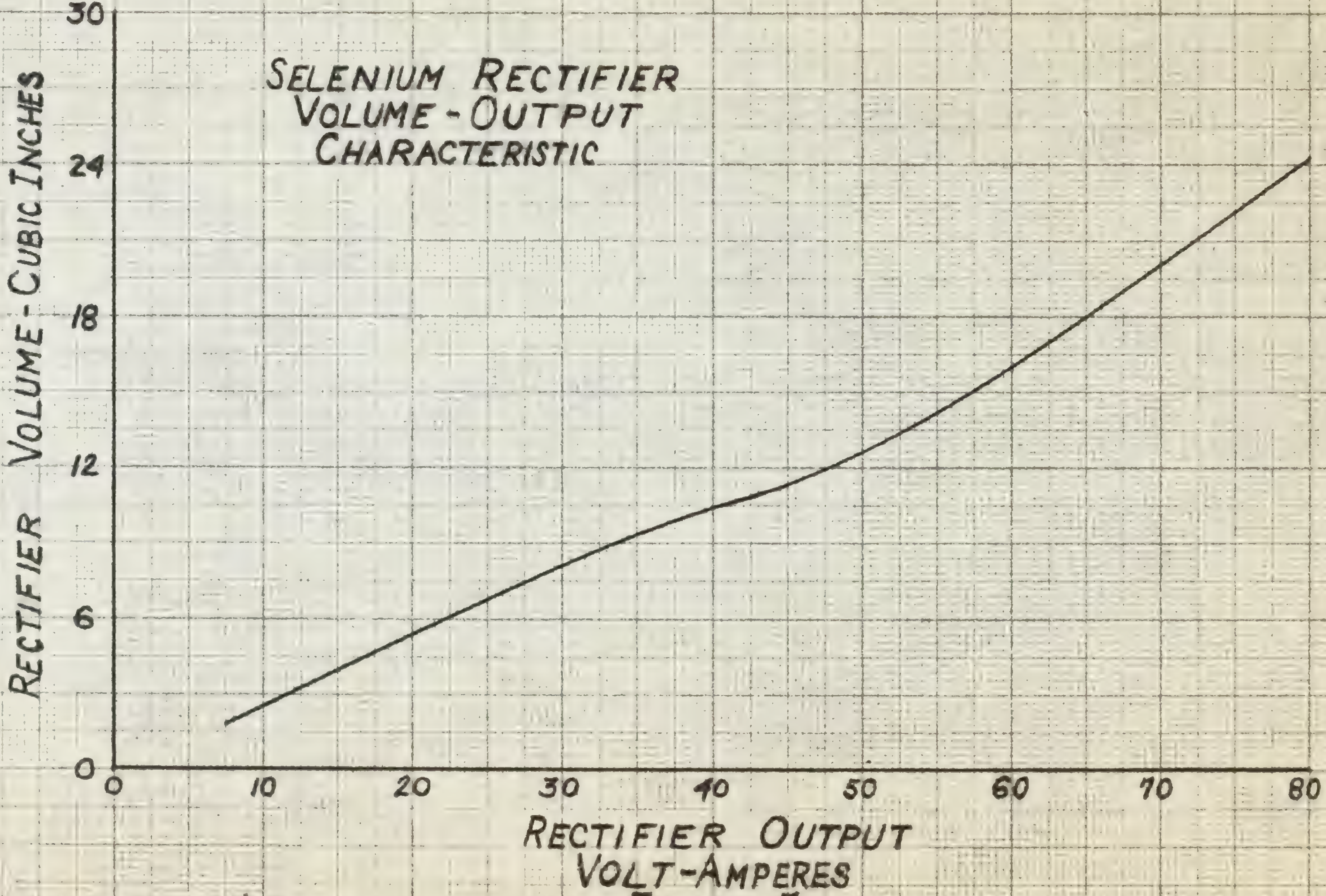
SELENIUM RECTIFIER
VOLUME - OUTPUT
CHARACTERISTIC
RECTIFIER VOLUME - CUBIC INCHES
0
6
12
18
24
30
0
10
20
30
40
50
60
70
80
RECTIFIER OUTPUT
VOLT-AMPERES

FIGURE 7.

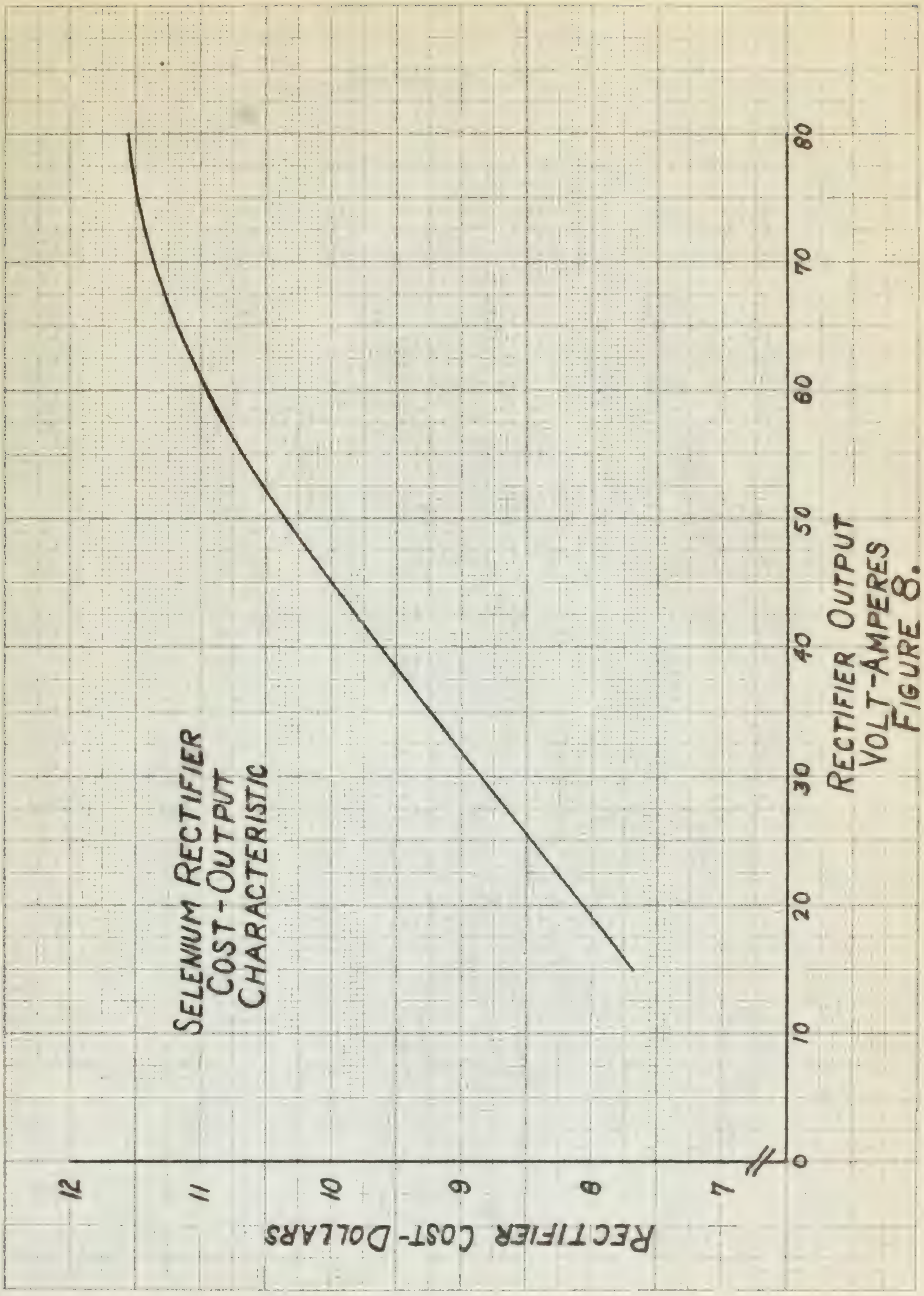

FIGURE 8.

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the 60-cycle alternating current magnet of section B-1. For comparison, it is assumed that the direct current magnet for the same pull would require the same volt-amperes, would weigh one-half as much as and occupy half the volume of the alternating current magnet. With these assumptions and with data from Figures 6 and 7, the 60-cycle alternating current and the direct current solutions may be compared.

	A.C. Magnet	D.C. Magnet	Selenium Rectifier	Total
A.C. Solution				
Weight	22.5 ozs.			22.5 ozs.
Space	4.1 cu.in.			4.1 cu.in.
D.C. Solution				
Weight		11.25 ozs.	5.6 ozs.	16.85 ozs.
Space		2.05 cu.in.	4.2 cu.in.	6.25 cu.in.

TABLE III

These figures are approximate, but the direct current solution has certainly been given no advantage; the rectifier data used are for standard units and the volt-ampere consumption of the direct current magnet has been taken to equal that of the alternating current magnet in selecting the rating of the rectifier.

It will be possible qualitatively to extend this comparison at 15 v.a. over the range of interest from 5 to 30 v.a. if the volume of the 60-cycle magnet can be found as a function of volt-ampere consumption. From

$$E = \sqrt{2}\,\pi f N \Phi_m \times 10^{-8} \qquad (1)$$

and

$$\Phi = M M F / R \qquad (10)$$

may be derived

$$V = K_3 \frac{\mu_e}{B_m^2} E I \qquad (41)$$

where V = volume, mean length times area

and $K_3 = \frac{.4}{\sqrt{2} \times 60 \times 10^{-3}}$

It has been shown in Figure 3 that for a given core material, μ_e increases with length. B_m is a function of the core material, not of the circuit dimensions and if it is assumed that the Type 11C relay was designed to operate at, or near maximum a.c. permeability, B_m may be considered to be constant. In any event, B_m varies over a small range for a given core material and is one of the first variables fixed in a design problem*. It follows immediately, therefore, that for good design (increasing μ_e as volume is increased), the total core volume would vary at least with the first power of volt ampere consumption.

Since the weight and space characteristics of the selenium rectifier vary about linearly with volt-ampere output, the relation between the alternating current and direct current solutions found in the preceding example may be considered to hold over the entire range of interest.

It is here considered, therefore, that the direct current solution retains its weight advantage and the possibility of a space disadvantage when compared with a 60-cycle magnet over the range of volt-ampere consumption from 6 to 80 v.a. If an efficient 400-cycle magnet had been compared

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*See page 247, "Electromagnetic Devices", H. C. Roters, John Wiley & Sons, Inc. 1941

with the direct current solution, the direct current weight advantage would have been greater and the space disadvantage less than indicated in the 60-cycle comparison.

Regarding economics, it is not possible to draw a positive conclusion. The 400-cycle magnet itself would undoubtedly be more expensive than a direct current magnet to exert the same pull. The running expense of the alternating current magnet would also be greater. The need for a selenium rectifier for use with the direct current magnet would work to offset these alternating current disadvantages.

Summarizing, on the basis of equal reliability, smaller weight, no volt-ampere limitation, a less serious contactor problem and less sustained power consumption, the author favors the direct current solution. He feels the space and cost considerations are still questionable, but even if they definitely indicated a 400-cycle solution, these considerations alone would not be strong enough to outweigh the advantages listed above for the direct current solution.

and

$$\Phi = M M F / \mathcal{R} \qquad (10)$$

may be derived

$$V = K_3 \frac{\mu_e}{B_m^2} E I \qquad (41)$$

where V = volume, mean length times area

and $K_3 = \frac{.4}{\sqrt{2} \times 60 \times 10^{-3}}$

It has been shown in Figure 3 that for a given core material, μ_e increases with length. B_m is a function of the core material, not of the circuit dimensions and if it is assumed that the Type 110 relay was designed to operate at, or near maximum a.c. permeability, B_m may be considered to be constant. In any event, B_m varies over a small range for a given core material and is one of the first variables fixed in a design problem*. It follows immediately, therefore, that for good design (increasing μ_e as volume is increased), the total core volume would vary at least with the first power of volt ampere consumption.

Since the weight and space characteristics of the selenium rectifier vary almost linearly with volt-ampere output, the relation between the alternating current and direct current solutions found in the preceding example may be considered to hold over the entire range of interest.

It is here considered, therefore, that the direct current solution retains its weight advantage and the possibility of a space disadvantage when compared with a 60-cycle magnet over the range of volt-ampere consumption from 5 to 25 v.a. If an efficient 400-cycle magnet had been compared

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*See page 247, "Electromagnetic Devices", H. C. Roters, John Wiley & Sons, Inc. 1941

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VITA

Charles Francis Helme, Jr. was born on July 25, 1919 at Pittston, Pennsylvania. He attended various schools in that state until 1936 when he was graduated from the Sayre High School, Sayre, Pennsylvania. He then attended the University of Pennsylvania where he began a course in Business Administration; in 1938 he entered the United States Naval Academy and was graduated with a degree of Bachelor of Science in Electrical Engineering in June 1942.

In July 1942, Ensign Helme began a tour of sea duty in destroyers which lasted until 1945. At that time he was selected for graduate work in Ordnance Engineering beginning at the Navy Post Graduate School, Annapolis, Maryland. In September of 1947, he entered the Graduate Department of the School of Engineering in Electrical Engineering at the Johns Hopkins University.

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